Thermoelastic Mechanism for Logarithmic Slow Dynamics and Memory in Elastic Wave Interactions with Individual Cracks

Vladimir Zaitsev,[1] Vitalyi Gusev,[2] and Bernard Castagnede[2]

[1]*Institute of Applied Physics RAS, 46 Uljanova Street, Nizhny Novgorod, 603950, Russia*
[2]*Université du Maine, Avenue Olivier Messiaen, 72085, Le Mans, Cedex 09, France*

(Received 26 September 2002; published 18 February 2003)

Logarithmic-in-time slow dynamics has been found for individual cracks in a solid. Furthermore, this phenomenon is observed during both the crack acoustic conditioning and the subsequent relaxation. A thermoelastic mechanism is suggested which relates the log-time behavior to the essentially 2D character of the heating and cooling of the crack perimeter and inner contacts. Nonlinear perturbation of the contacts by a stronger (pump) wave causes either softening or hardening of the sample, and induces either additional absorption or transparency for a weaker (probe) acoustic wave depending on frequency of the latter.

DOI: 10.1103/PhysRevLett.90.075501 PACS numbers: 62.65.+k, 43.25.+y, 62.20.Mk, 91.60.Lj

Introduction.—For many materials with granular and imperfect structure, effects with slow dynamics that are logarithmic in time are remarkably common. Examples are the accommodation and aftereffect in magnetic materials [1], creep phenomena in metals [2], relaxation of rocks after acoustic stressing [3], memory and aging of granular materials [4], and irreversible temperature rise in glasses [5]. This universal log-time behavior is usually attributed to the complexity of dynamical processes in systems with wide distribution of energy barriers that should be overcome for the relaxation of the perturbed ensemble of the mechanical bonds [2–4]. The initial rapid relaxation is due to quick thermal flipping over small-energy barriers. As their numbers are thereby exhausted, the relaxation rate falls, since thermal fluctuations capable of activating the higher barriers are exponentially rare. Note that logarithmic dynamics arises only for a suitable wide spectrum of activation energies, the origin of which and its relation to the material microstructure remains unknown [2,3,5].

Recently, besides solids with numerous microdefects [3], observations of slow relaxation and memory effects were reported for interaction of ultrasonic waves with a single crack [6]. For parametric generation of sub- and superharmonics of the ‘‘reading-out’’ elastic wave, the threshold amplitudes remained perturbed up to minutes after activating the crack by another intense wave. However, for linearly transmitted or reflected wave components at the fundamental frequency, evidence of slow dynamics of the individual crack was not found [6].

In the present Letter, we report observations of both the relaxation on the scale of the acoustic period and slow relaxation effects in the interaction of elastic waves with individual cracks. Our approach allowed us to carefully study the slow temporal evolution of both the sample elastic and dissipative properties connected to perturbation of the crack by acoustic loading. For the first time we observed the logarithmic-in-time slow dynamics of individual cracks subjected to ‘‘conditioning’’ by an elastic wave. Moreover, the logarithmic behavior was found *during both the crack acoustic conditioning and the subsequent relaxation*. We will show that the ‘‘instantaneous’’ effects (of material softening or hardening and induced transparency or absorption) as well as the slow dynamics, revealed in the wave-crack interaction, can be consistently explained by the (i) direct acoustic perturbation of inner contacts via the nonlinear effect of strain rectification and (ii) the contact slow ‘‘breathing’’ due to acoustically induced thermal microstrains within the crack. Below we point out the main experimentally found features and present some theoretical arguments confirming the formulated idea.

Experimental technique.—In this study the experimental technique and setup described in [7] was supplemented by measurements of the slow evolution of the sample properties. We observed the temporal behavior of resonance peaks for a weak (probe) longitudinal wave with typical strain $\varepsilon < 10^{-8}$ in glass rods subjected to action of another conditioning (pump) wave with typical strains up to $\varepsilon \sim 10^{-6}$–10^{-5}. The rods contained 1–3 thermally produced cracks. By measuring the positions, widths, and amplitudes of the probe-wave resonances we could simultaneously follow the variations in the elastic and the dissipative sample properties. The maximum amplitudes and inverse widths of a resonance peak are proportional to the current quality factor $Q = \pi/\theta$, where θ is the decrement. For a few peaks, whose shape was closest to a Lorentzian, we checked that the estimates of the Q-factor variation by the peak widening and by the amplitude variation δA were consistent. Afterwards we used the amplitude method, which gave a much better accuracy, within $\delta A/A_0 \sim 10^{-2}$, sufficient to study the pump-induced variations in the Q factor that reached 10% or even more.

Fast rectification effects.—In order to better comprehend our observations it is important first to recall the following. For cracks of millimeter-size L, the maximal normal and lateral interface displacement $\sim\varepsilon L$ for wave strains $\varepsilon \sim 10^{-8}$ is of subatomic scale [7]. Because of this, the frictional and hysteretic losses are not yet activated for the probe wave. However, losses due to the thermoelastic coupling may be very efficient, especially at narrow inner contacts between the crack lips. For contacts of width $l \ll L$, the theoretical analysis of the losses [7] indicates a relaxation peak at $\omega = \omega_l \approx D/l^2$, where D is the temperature diffusion coefficient. For the whole crack, its relaxation frequency ω_L is determined by its scale $L \gg l$, so that $\omega_L \approx D/L^2 \ll \omega_l$. For most rocks and glasses, typical ω_L for a millimeter-scale crack corresponds to fractions of a cycle/s, whereas for micrometer-width inner contacts the respective frequency may lie from kHz up to MHz band. The latter readily explains strongly increased small-amplitude (linear) acoustic dissipation in the kHz range at millimeter-scale cracks [7]. Further, since the local separation of crack lips in the contact vicinity may be orders of magnitude smaller than the average crack opening, waves with moderate strain $\varepsilon \sim 10^{-6}$–10^{-5} may easily perturb the inner microcontacts (and even cause their clapping), although the average crack opening remains hardly perturbed. For example, for a contact whose initial strain ε_0 and local applied stress σ_0 are related by the Hertz-type dependence $\varepsilon \propto \sigma^{2/3}$, the superimposed wave with oscillatory stress σ_w comparable to σ_0 may significantly reduce period-averaged contact strain $\langle\varepsilon\rangle$, as schematically illustrated in Fig. 1. It is worth mentioning that similar rectification (demodulation) effects are well documented in nonlinear acoustics of the interfaces [8] and provide the basis for the ultrasonic force mode in atomic force microscopy [9]. The resultant reduction of the averaged contact width shifts the maximum of the thermoelastic losses at the contact to a higher frequency ω_l (see inset of Fig. 1). This should cause an *increase* in the quality factor of the sample resonances located lower than ω_l in the frequency domain, but simultaneously *decrease* the quality factor for the resonances located higher than ω_l.

In order to demonstrate the latter effect, among the fabricated samples we succeeded to choose one in which a thermally induced crack exhibited such a behavior.

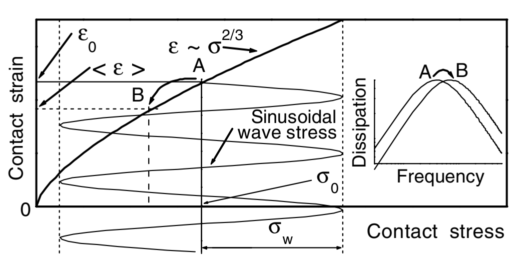

FIG. 1. Schematically shown softening of the contact by oscillating stress due to loading-unloading asymmetry. Initial static equilibrium $A = (\sigma_0, \varepsilon_0)$, and the perturbed time-averaged position $B = (\langle\sigma\rangle, \langle\varepsilon\rangle)$.

Figure 2 shows records of the probe-wave resonance peak below 4 kHz, which exhibited a pronounced *increase* in the quality factor under the pump-wave action, whereas the next mode above 10 kHz and a dozen or so other peaks within the observable band up to 100 kHz exhibited a *decrease* in the quality factor. In contrast to the opposite trends in the dissipation, the resonant frequencies for all peaks exhibited a *consistent decrease,* as should be expected owing to the time-average softening of the microcontact(s) induced by the pump wave.

Further increase of the oscillation amplitude, $\sigma_w > \sigma_0$, transfers the contacts to the clapping regime, in which the averaged stiffness and size of near-Hertzian contacts begin to increase again. In this case both the decrement of the probe-peak and its position in the frequency domain may become nonmonotonous functions of the pump-wave amplitude. Resonance curves shown in Fig. 3 demonstrate such a nonmonotonous dependence on the pump-amplitude both for the amplitude (that is dissipation) and the frequency shift of one of the probe-wave peaks. The inset shows the variation in the averaged strain $\langle\varepsilon\rangle$ simulated for a Hertzian contact as a function of normalized oscillating stress σ_w/σ_0, which indicates similar nonmonotonous behavior.

Elasticity/dissipation slow-dynamics.—Further investigation of the aforementioned crack-induced effects revealed that both the dissipation and the resonance frequencies exhibited pronounced *slow dynamics.* The characteristic magnitudes of the slow drifts were sometimes comparable to instantaneous variations produced due to the pump-rectification effect. To study the

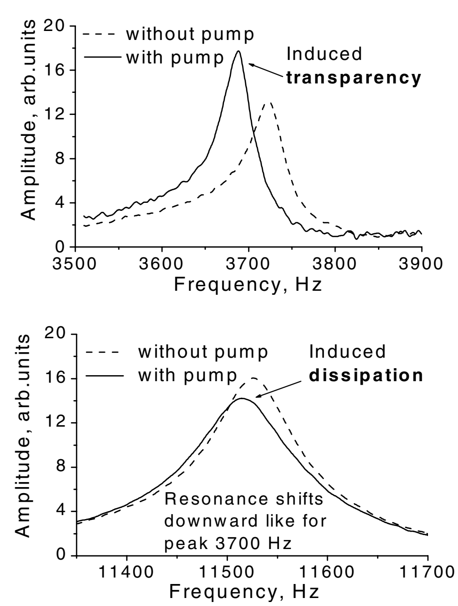

FIG. 2. Examples of the complementary pump-induced *decrease* in dissipation for the probe wave around 3.7 kHz and *increase* in dissipation for the probe wave around 11.5 kHz. The resonance frequency decreases in both cases. Pump wave strain is $\varepsilon \sim 10^{-6}$.

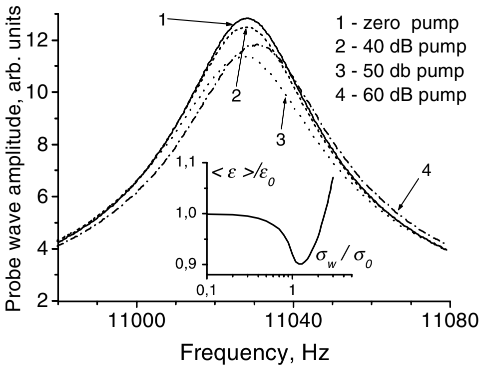

FIG. 3. Example of nonmonotonous amplitude dependence of the pump-induced variations in dissipation and the probe wave resonance frequency. The inset shows the simulated averaged contact strain plotted against the pump-stress amplitude.

phenomenon we documented the drifts in the probe-wave resonance curves with 15 s intervals required to store a single record. Figure 4(a) presents examples of variations in the resonance peak amplitudes plotted against the logarithm of time elapsed after both switching on or switching off the pump wave. Figure 4(b) shows similar slow variations $\delta f/f_0$ of the resonance frequency. Thus the observed logarithmic slow dynamics produced in the sample dissipation and elasticity by *individual cracks*

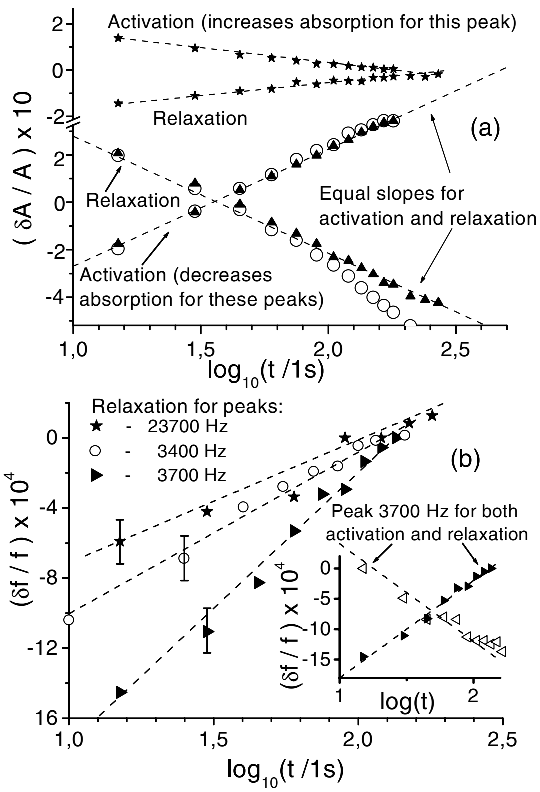

FIG. 4. Examples of the log-time dependence of the dissipation (a) and the resonance frequency shift (b) for different peaks.

subjected to acoustic loading is strikingly similar to the slow relaxation effects found for rocks with inherent numerous defects [3]. A remarkable newly revealed feature is that the log-time behavior during the crack conditioning exhibits exactly *the same slope* (rate) as for the postconditioning relaxation.

In the context of the above considered mechanism of the crack/contact influence on the probe-wave resonances, explanation for the slow logarithmic relaxation arrives rather naturally from the dynamics of thermal conduction in a cylindrical geometry. This geometry is specific for both the crack perimeter and the elongated inner microcontacts, which, as it is argued in detail in [7], produce the main contribution to the dissipation and hence undergo local acoustic heating. The contact state may be strongly perturbed by nanoscale absolute distortions at the crack interfaces. As discussed above, such averaged distortions for millimeter-scale cracks are produced by rectification of the pump oscillations with strains $\varepsilon \sim 10^{-6}$–10^{-5} and are responsible for the instantaneous effects. Alternatively, similar distortions in the crack may be expected as a result of temperature inhomogeneities about $\Delta T \sim 0.1$–1 K. For a typical thermal expansion coefficient $\alpha \sim 3 \times 10^{-6}$ K^{-1} and crack size $L \sim 3 \times 10^{-3}$ m, the thermoelastic displacement is estimated as $\alpha L \Delta T \sim 10^{-9}$–$10^{-8}$ m, which is comparable with the instantaneous rectification effect. Note that direct infrared imaging of acoustically induced heating of several degrees at the stress-concentration areas at crack tips and lips is available [10]. The logarithmic slowing of the temperature rise (in the case of conditioning) in 2D geometry is due to the diminishing of the heat flow, which in turn is caused by the increase in the spatial scale of the heated region and the decrease of the temperature gradients with time. Indeed, for a steplike (in time) cylindrical thermal source $Q(r, t)$ localized in the area of a radius $r \leq l$, the 2D equation for heat conduction $\partial T/\partial t - D\Delta_{\perp} T = Q/(\rho C)$ yields an asymptotically logarithmic law for the temperature increase ΔT in the source vicinity:

$$\Delta T \approx \frac{Q_F(k=0)}{4\pi\rho C D} \ln\frac{t}{l^2/D}, \qquad \text{for } t \gg l^2/D = \omega_l^{-1}. \tag{1}$$

Here ρ and C are the material density and specific heat, and $Q_F(k)$ is the spatial Fourier transform of the source. For the subsequent cooling after switching off the source Q at time $t = t_0$, there is also a log-time approximate solution,

$$\Delta T \approx \frac{Q_F(k=0)}{4\pi\rho C D}\left[\ln\frac{t_0}{l^2/D} - \ln\frac{(t-t_0)}{l^2/D}\right], \tag{2}$$

valid for $l^2/D \ll t - t_0 \leq t_0$ and indicating *the same slope* as in (1), which is predetermined by the temperature spatial distribution produced by the initial heating. We already pointed out that equal slopes for the conditioning

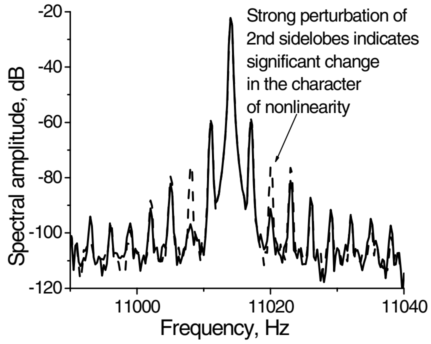

FIG. 5. Examples of cross-modulation spectra just after intensive conditioning (solid line) and 5 min later (dashed line). Pump wave carrier and modulation frequencies were 3.7 kHz and 3 Hz, respectively.

and relief were observed in the experiments (see Fig. 4), which is a strong argument supporting the heating-cooling mechanism. Indeed, the state of the stress-concentration areas is essentially different at rest and during intensive acoustic activation that causes contact clapping and the rectification effect. Therefore, if the dominant contribution to the observed slow drifts were connected to the mechanism [2–4] of gradual rupture or restoration of large ensembles of local bonds, then different slopes should be expected for vigorously vibrating and relaxing contact areas.

The thermal conductivity mechanism in the log-time behavior is likely to play a dominant role for time scales $\omega_l^{-1} < t < \omega_L^{-1}$ ranging within 5–7 orders of magnitude, in which the thermal conductivity keeps its quasi-2D character. For thermal diffusivity $D \sim 10^{-7}$–10^{-6} m^2 s^{-1} typical of glasses and rocks, and micron-scale width l and millimeter-scale lengths L, the log-time behavior may persist in the interval from 10^{-3}–10^{-5} s up to several minutes, which agrees with our observations. Note, finally, that in the crack-containing sample, the Q factor was reduced from values of about $Q_0 \sim 250$–300 in the reference (intact) sample to $Q_1 \sim$ 130–160, corresponding to the crack-induced decrement $\Delta\theta = \pi(Q_1^{-1} - Q_0^{-1}) \sim 2 \times 10^{-2}$. Since this additional dissipation causes heating of the stress-concentration regions of total length $\sim L$, then the thermal source $Q_F(k = 0)$ in (1) can be estimated as $Q_F(k = 0) \sim \Delta\theta W f/L$, where f is the pump-wave frequency and W is the pump-wave elastic energy accumulated in the sample (a glass rod 8 mm in radius and 30 cm in length). Thus, for pump strains $\varepsilon \sim 10^{-6}$–10^{-5}, Eq. (1) yields $\Delta T \sim$ 0.1–10 K at our observation times, in agreement with [10] and the above assumed magnitude of ΔT.

Evidence for memory in nonlinearity.—Attempting to reveal slow dynamics in nonlinear crack-induced effects (for purposes of comparison with [6]), we faced a problem that the nonlinearly excited harmonics were essentially influenced by the system resonances, whose slow dynamics strongly masked possible memory in the nonlinear properties. We finally succeeded in observing a very clear manifestation of memory for the nonlinear effect of the cross modulation [7] of the probe wave by a slowly modulated pump wave. Figure 5 presents the modulation spectra obtained immediately after a few minutes of intensive conditioning of the sample and after a 5 min pause. Since the stronger modulating wave also acted as a pump, after the pause we switched it on again for only the several seconds required to obtain the spectrum. There is a remarkable difference in the second side lobe amplitude indicating a significant change in the character of the crack-induced nonlinearity. In contrast, the amplitudes of the fundamental line and other side lobes are much more weakly perturbed, which assures that the whole resonance curve remained almost the same.

Conclusion.—The technique applied in this study revealed memory and slow-dynamics effects induced by individual cracks both in the sample nonlinearity and in the linear elasticity and absorption. The observed variety of instantaneous and slow-dynamics effects is consistently explained by the suggested mechanism based on a few well established features of the acoustic wave-crack interaction, implying slowly varying thermal micro-strains. These findings provide a new physical insight into the origin of the logarithmic slow dynamics in imperfect solids and should stimulate both new interpretations of known pertinent data and new experiments. Besides, our models may also be relevant to the effects of the amplitude-dependent absorption or transparency for the acoustic waves observed in other systems, in particular, in sand and sandstone ([11] and references therein).

The study was supported in part by the RFBR Grant No. 02-02-16237 and DGA Contract No. 00.34.026.

[1] L. J. Swartzendruber, L. H. Bennett, F. Vaida, and E. Della Torre, Physica (Amsterdam) **233B**, 324 (1997).
[2] A. H. Cottrell, Philos. Mag. Lett. **75**, 301 (1997).
[3] J. A. Ten Cate, E. Smith, and R. Guyer, Phys. Rev. Lett. **85**, 1020 (2000).
[4] H. M. Jager, C.-H. Liu, and S. R. Nagel, Phys. Rev. Lett. **62**, 40 (1989); L Bocquet *et al.*, Nature (London) **396**, 735 (1998).
[5] O. B. Wright and W. A. Philips, Philos. Mag. B **50**, 63 (1984).
[6] I. Yu. Solodov and B. Korshak, Phys. Rev. Lett. **88**, 014303 (2002).
[7] V. Yu. Zaitsev, V. Gusev, and B. Castagnede, Ultrasonics **40**, 627 (2002); Phys. Rev. Lett. **89**, 105502 (2002).
[8] I. Yu. Solodov, Ultrasonics **36**, 383 (1998).
[9] O. Kolosov and K. Yamanaka, Jpn. J. Appl. Phys. **32**, L1095 (1993).
[10] L. D. Favro *et al.*, Rev. Sci. Instrum. **71**, 2418 (2000).
[11] V. E. Nazarov, A. V. Radostin, and Yu. A. Stepanyants, Acoust. Phys. **47**, 733 (2001); V. E. Nazarov, A. V. Radostin, and I. A. Soustova, Acoust. Phys. **48**, 76 (2002).